

**GCSE (9 – 1) Mathematics Two Year Scheme of Work**
**Foundation Tier**

Unit		Title	Estimated hours
<u>1</u>	<u>a</u>	Integers and place value	4
	<u>b</u>	Decimals	3
	<u>c</u>	Indices, powers and roots	5
	<u>d</u>	Factors, multiples and primes	4
<u>2</u>	<u>a</u>	Algebra: the basics	6
	<u>b</u>	Expressions and substitution into formulae	5
<u>3</u>	<u>a</u>	Tables, charts and graphs	11
	<u>b</u>	Pie charts	3
	<u>c</u>	Scatter graphs	4
<u>4</u>	<u>a</u>	Fractions, decimals and percentages	7
	<u>b</u>	Percentages	6
<u>5</u>	<u>a</u>	Equations and inequalities	9
	<u>b</u>	Sequences	5
<u>6</u>	<u>a</u>	Properties of shapes, parallel lines and angle facts	7
	<u>b</u>	Interior and exterior angles of polygons	4
<u>7</u>		Statistics, sampling and the averages	7
<u>8</u>		Perimeter, area and volume	10
<u>9</u>	<u>a</u>	Real-life graphs	8
	<u>b</u>	Straight-line graphs	6
<u>10</u>		Transformations	11
<u>11</u>	<u>a</u>	Ratio	4
	<u>b</u>	Proportion	5
<u>12</u>		Right-angled triangles: Pythagoras and trigonometry	5
<u>13</u>		Probability	12
<u>14</u>		Multiplicative reasoning	7
<u>15</u>	<u>a</u>	Plans and elevations	5
	<u>b</u>	Constructions, loci and bearings	7
<u>16</u>	<u>a</u>	Quadratic equations: expanding and factorising	5
	<u>b</u>	Quadratic equations: graphs	4
<u>17</u>		Circles, cylinders, cones and spheres	6
<u>18</u>	<u>a</u>	Fractions and reciprocals	5
	<u>b</u>	Indices and standard form	5
<u>19</u>	<u>a</u>	Similarity and congruence in 2D	7
	<u>b</u>	Vectors	7
<u>20</u>		Rearranging equations, graphs of cubic and reciprocal functions and simultaneous equations	5

## UNIT 1: Number, powers, decimals, HCF and LCM, roots and rounding

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### SPECIFICATION REFERENCES

- N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
- N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
- N5 apply systematic listing strategies
- N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- N7 calculate with roots and with integer and with integer indices
- N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures);

### PRIOR KNOWLEDGE

Students will have an appreciation of place value, and recognise even and odd numbers.

Students will have knowledge of using the four operations with whole numbers.

Students should have knowledge of integer complements to 10 and to 100.

Students should have knowledge of strategies for multiplying and dividing whole numbers by 2, 4, 5, and 10.

Students should be able to read and write decimals in figures and words.

### KEYWORDS

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd

<b>1a. Integers and place value (N1, N2, N3, N4, N14, N15)</b>		<b>Estimated 4 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use and order positive and negative numbers (integers) and decimals; use the symbols <math>&lt;</math>, <math>&gt;</math> and understand the <math>\neq</math> symbol;</li> <li>• Add, subtract, multiply and divide positive and negative numbers (integers);</li> <li>• Recall all multiplication facts to <math>10 \times 10</math>, and use them to derive quickly the corresponding division facts;</li> <li>• Multiply or divide any number by powers of 10;</li> <li>• Use brackets and the hierarchy of operations (not including powers);</li> <li>• Round numbers to a given power of 10;</li> <li>• Check answers by rounding and using inverse operations.</li> </ul>	<ul style="list-style-type: none"> <li>• Given 5 digits, what are the largest or smallest answers when subtracting a two-digit number from a three-digit number?</li> <li>• Use inverse operations to justify answers, e.g. <math>9 \times 23 = 207</math> so <math>207 \div 9 = 23</math>.</li> <li>• Check answers by rounding to nearest 10, 100, or 1000 as appropriate, e.g. <math>29 \times 31 \approx 30 \times 30</math></li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Missing digits in calculations involving the four operations</li> <li>• Questions such as: Phil states <math>3.44 \times 10 = 34.4</math> and Chris states <math>3.44 \times 10 = 34.40</math>. Who is correct?</li> <li>• Show me another number with 3, 4, 5, 6, 7 digits that includes a 6 with the same value as the "6" in the following number 36, 754</li> </ul>	<ul style="list-style-type: none"> <li>• Stress the importance of knowing the multiplication tables to aid fluency.</li> <li>• Students may write statements such as <math>150 - 210 = 60</math>.</li> </ul>	

Notes	Suggested Activities
<ul style="list-style-type: none"><li>• Much of this unit will have been encountered by students in previous Key Stages, meaning that teaching time may focus on application or consolidation of prior learning.</li><li>• Particular emphasis should be given to the importance of students presenting their work clearly.</li><li>• Formal written methods of addition, subtraction and multiplication work from right to left, whilst formal division works from left to right.</li><li>• Any correct method of multiplication will still gain full marks, for example, the grid method, the traditional method, Napier's bones.</li><li>• Negative numbers in real life can be modelled by interpreting scales on thermometers using F and C.</li><li>• Encourage the exploration of different calculation methods.</li><li>• Students should be able to write numbers in words and from words as a real-life skill.</li></ul>	

<b>1b. Decimals (N1, N2, N3, N13, N15)</b>		<b>Estimated 3 hours</b>
<p><b>Objectives</b></p> <p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use decimal notation and place value;</li> <li>• Identify the value of digits in a decimal or whole number;</li> <li>• Compare and order decimal numbers using the symbols <math>&lt;</math>, <math>&gt;</math>;</li> <li>• Understand the <math>\neq</math> symbol (not equal);</li> <li>• Write decimal numbers of millions, e.g. <math>2\ 300\ 000 = 2.3</math> million;</li> <li>• Add, subtract, multiply and divide decimals;</li> <li>• Multiply or divide by any number between 0 and 1;</li> <li>• Round to the nearest integer;</li> <li>• Round to a given number of decimal places and significant figures;</li> <li>• Estimate answers to calculations by rounding numbers to 1 significant figure;</li> <li>• Use one calculation to find the answer to another.</li> </ul>	<p><b>Possible Success Criteria</b></p> <ul style="list-style-type: none"> <li>• Use mental methods for <math>\times</math> and <math>\div</math>, e.g. <math>5 \times 0.6</math>, <math>1.8 \div 3</math>.</li> <li>• Solve a problem involving division by a decimal (up to 2 decimal places).</li> <li>• Given <math>2.6 \times 15.8 = 41.08</math>, what is <math>26 \times 0.158</math>? What is <math>4108 \div 26</math>?</li> <li>• Calculate, e.g. <math>5.2</math> million + <math>4.3</math> million.</li> </ul>	
<p><b>Opportunities for reasoning/problem solving</b></p> <ul style="list-style-type: none"> <li>• Problems involving shopping for multiple items, such as: Rob purchases a magazine costing £2.10, a newspaper costing 82p and two bars of chocolate. He pays with a £10 note and gets £5.40 change. Work out the cost of one bar of chocolate.</li> <li>• When estimating, students should be able to justify whether the answer will be an overestimate or underestimate.</li> </ul>	<p><b>Common Misconceptions</b></p> <ul style="list-style-type: none"> <li>• Significant figures and decimal place rounding are often confused.</li> <li>• Some students may think <math>35\ 877 = 36</math> to two significant figures.</li> </ul>	
<p><b>Notes</b></p> <ul style="list-style-type: none"> <li>• Practise long multiplication and division, use mental maths problems with decimals such as 0.1, 0.001.</li> <li>• Amounts of money should always be rounded to the nearest penny.</li> </ul>	<p><b>Suggested Activities</b></p>	

<b>1c. Indices, powers and roots (N6, N7)</b>		<b>Estimated 5 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Find squares and cubes;</li> <li>• recall integer squares up to <math>10 \times 10</math> and the corresponding square roots;</li> <li>• understand the difference between positive and negative square roots;</li> <li>• recall the cubes of 1, 2, 3, 4, 5 and 10;</li> <li>• Use index notation for squares and cubes;</li> <li>• Recognise powers of 2, 3, 4, 5;</li> <li>• Evaluate expressions involving squares, cubes and roots:</li> <li>• add, subtract, multiply and divide numbers in index form;</li> <li>• cancel to simplify a calculation;</li> <li>• Use index notation for powers of 10, including negative powers;</li> <li>• Use the laws of indices to multiply and divide numbers written in index notation;</li> <li>• Use brackets and the hierarchy of operations with powers inside the brackets, or raising brackets to powers;</li> <li>• Use calculators for all calculations: positive and negative numbers, brackets, square, cube, powers and roots, and all four operations.</li> </ul>	<ul style="list-style-type: none"> <li>• What is the value of <math>2^3</math>?</li> <li>• Evaluate <math>(2^3 \times 2^5) \div 2^4</math>.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Problems such as: What two digit number is special because adding the sum of its digits to the product of its digits gives me my original number?</li> </ul>	<ul style="list-style-type: none"> <li>• The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.</li> <li>• <math>10^3</math>, for example, is interpreted as <math>10 \times 3</math>.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Pupils need to know how to enter negative numbers into their calculator.</li> <li>• Use the language of 'negative' number and not minus number to avoid confusion with calculations.</li> <li>• Note that the students need to understand the term 'surd' as there will be occasions when their calculator displays an answer in surd form, for example, <math>4\sqrt{2}</math>.</li> </ul>	<ul style="list-style-type: none"> <li>•</li> </ul>	

<b>1d. Factors, multiples and primes (N4, N5)</b>		<b>Estimated 4 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>List all three-digit numbers that can be made from three given integers;</li> <li>Recognise odd, even and prime (two digit) numbers;</li> <li>Identify factors and multiples and list all factors and multiples of a number systematically;</li> <li>Find the prime factor decomposition of positive integers and write as a product using index notation;</li> <li>Find common factors and common multiples of two numbers;</li> <li>Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors: include finding LCM and HCF given the prime factorisation of two numbers;</li> <li>Understand that the prime factor decomposition of a positive integer is unique – whichever factor pair you start with – and that every number can be written as a product of two factors;</li> <li>Solve simple problems using HCF, LCM and prime numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Given the digits 1, 2 and 3, find how many numbers can be made using all the digits.</li> <li>Convince me that 8 is not prime.</li> <li>Understand that every number can be written as a unique product of its prime factors.</li> <li>Recall prime numbers up to 100.</li> <li>Understand the meaning of prime factor.</li> <li>Write a number as a product of its prime factors.</li> <li>Use a Venn diagram to sort information.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>Students should be able to provide convincing counter-arguments to statements concerning properties of stated numbers, i.e. Sharon says 108 is a prime number. Is she correct?</li> <li>Questions that require multiple layers of operations such as: Pam writes down one multiple of 9 and two different factors of 40. She then adds together her three numbers. Her answer is greater than 20 but less than 30. Find three numbers that Jan could have written down.</li> </ul>	<ul style="list-style-type: none"> <li>1 is a prime number.</li> <li>Particular emphasis should be made on the definition of 'product' as multiplication as many students get confused and think it relates to addition.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>Use a number square to find primes (Eratosthenes sieve).</li> <li>Using a calculator to check factors of large numbers can be useful.</li> <li>Students need to be encouraged to learn squares from <math>2 \times 2</math> to <math>15 \times 15</math> and cubes of 2, 3, 4, 5 and 10 and corresponding square and cube roots.</li> </ul>	<ul style="list-style-type: none"> <li></li> </ul>	

## UNIT 2: Expressions, substituting into simple formulae, expanding and factorising

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### SPECIFICATION REFERENCES

- N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
- N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- A1 use and interpret algebraic notation, including:
- $ab$  in place of  $a \times b$
  - $3y$  in place of  $y + y + y$  and  $3 \times y$
  - $a^2$  in place of  $a \times a$ ,  $a^3$  in place of  $a \times a \times a$ ,  $a^2b$  in place of  $a \times a \times b$
  - $\frac{a}{b}$  in place of  $a \div b$
  - coefficients written as fractions rather than as decimals
  - brackets
- A2 substitute numerical values into formulae and expressions, including scientific formulae
- A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors
- A4 simplify and manipulate algebraic expressions ... by:
- collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors ...
  - simplifying expressions involving sums, products and powers, including the laws of indices
- A5 understand and use standard mathematical formulae; rearrange formulae to change the subject
- A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- A7 where appropriate, interpret simple expressions as functions with inputs and outputs
- A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution

### PRIOR KNOWLEDGE

Students should have prior knowledge of some of these topics, as they are encountered at Key Stage 3:

- the ability to use negative numbers with the four operations and recall and use hierarchy of operations and understand inverse operations;
- dealing with decimals and negatives on a calculator;
- using index laws numerically.

### KEYWORDS

Expression, identity, equation, formula, substitute, term, 'like' terms, index, power, collect, substitute, expand, bracket, factor, factorise, linear, simplify

<b>2a. Algebra: the basics (N1, N3, A1, A3, A4)</b>		<b>Estimated 6 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use notation and symbols correctly;</li> <li>• Write an expression;</li> <li>• Select an expression/equation/formula/identity from a list;</li> <li>• Manipulate and simplify algebraic expressions by collecting 'like' terms;</li> <li>• Multiply together two simple algebraic expressions, e.g. <math>2a \times 3b</math>;</li> <li>• Simplify expressions by cancelling, e.g. <math>\frac{4x}{2} = 2x</math>;</li> <li>• Use index notation and the index laws when multiplying or dividing algebraic terms;</li> <li>• Understand the <math>\neq</math> symbol and introduce the identity <math>\equiv</math> sign;</li> </ul>	<ul style="list-style-type: none"> <li>• Simplify <math>4p - 2q + 3p + 5q</math>.</li> <li>• Simplify <math>z^4 \times z^3, y^3 \div y^2, (a^7)^2</math>.</li> <li>• Simplify <math>x^{-4} \times x^2, w^2 \div w^{-1}</math>.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Forming expressions and equations using area and perimeter of 2D shapes.</li> </ul>	<ul style="list-style-type: none"> <li>• Any poor number skills involving negatives and times tables will become evident.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Some of this will be a reminder from Key Stage 3.</li> <li>• Emphasise correct use of symbolic notation, i.e. <math>3 \times y = 3y</math> and not <math>y3</math> and <math>a \times b = ab</math>.</li> <li>• Use lots of concrete examples when writing expressions, e.g. 'B' boys + 'G' girls.</li> <li>• Plenty of practice should be given and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed.</li> </ul>		

**2b. Expressions and substitution into formula (A2, A4, A5, A6, A7, A21)****Estimated 5 hours**

<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"><li>• Multiply a single number term over a bracket;</li><li>• Write and simplify expressions using squares and cubes;</li><li>• Simplify expressions involving brackets, i.e. expand the brackets, then add/subtract;</li><li>• Argue mathematically to show algebraic expressions are equivalent;</li><li>• Recognise factors of algebraic terms involving single brackets;</li><li>• Factorise algebraic expressions by taking out common factors;</li><li>• Write expressions to solve problems representing a situation;</li><li>• Substitute numbers into simple algebraic expressions;</li><li>• Substitute numbers into expressions involving brackets and powers;</li><li>• Substitute positive and negative numbers into expressions;</li><li>• Derive a simple formula, including those with squares, cubes and roots;</li><li>• Substitute numbers into a (word) formula;</li></ul>	<ul style="list-style-type: none"><li>• Expand and simplify <math>3(t - 1)</math>.</li><li>• Understand <math>6x + 4 \neq 3(x + 2)</math>.</li><li>• Argue mathematically that <math>2(x + 5) = 2x + 10</math>.</li><li>• Evaluate the expressions for different values of <math>x</math>: <math>3x^2 + 4</math> or <math>2 \times 3</math>.</li></ul>
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>
<ul style="list-style-type: none"><li>• Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.</li></ul>	<ul style="list-style-type: none"><li>• <math>3(x + 4) = 3x + 4</math>.</li><li>• The convention of not writing a coefficient with a single value, i.e. <math>x</math> instead of <math>1x</math>, may cause confusion.</li><li>• Some students may think that it is always true that <math>a = 1</math>, <math>b = 2</math>, <math>c = 3</math>.</li><li>• If <math>a = 2</math> sometimes students interpret <math>3a</math> as <math>32</math>.</li><li>• Making mistakes with negatives, including the squaring of negative numbers.</li></ul>

Notes	Suggested Activities
<p>Provide students with lots of practice. This topic lends itself to regular reinforcement through starters in lessons. Use formulae from mathematics and other subjects, expressed initially in words and then using letters and symbols. Include substitution into the kinematics formulae given on the formula sheet, i.e. <math>v = u + at</math>, <math>v^2 - u^2 = 2as</math>, and <math>s = ut + \frac{1}{2} at^2</math>.</p>	

## UNIT 3: Drawing and interpreting graphs, tables and charts

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### SPECIFICATION REFERENCES

- G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
- G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
- G15 measure line segments and angles in geometric figures ...
- S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
- S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
- appropriate graphical representation involving discrete, continuous and grouped data
  - appropriate measures of central tendency (... mode and modal class) and spread (range, including consideration of outliers)
- S5 apply statistics to describe a population
- S6 use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

### PRIOR KNOWLEDGE

Students should be able to read scales on graphs, draw circles, measure angles and plot coordinates in the first quadrant, and know that there are 360 degrees in a full turn and 180 degrees at a point on a straight line.

Students should have experience of tally charts.

Students will have used inequality notation.

Students must be able to find the midpoint of two numbers.

Students should be able to use the correct notation for time using 12- and 24-hour clocks.

### KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, scatter graph, line of best fit, correlation, positive, negative, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate

**3a. Tables, charts and graphs (G14, S2, S4, S5)****Estimated 11 hours**

<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"><li>• Use suitable data collection techniques (data to be integer and decimal values);</li><li>• Design and use data-collection sheets for grouped, discrete and continuous data, use inequalities for grouped data, and introduce <math>\leq</math> and <math>\geq</math> signs; Sort, classify and tabulate data, both discrete and continuous quantitative data, and qualitative data; Extract data from lists and tables;</li><li>• Use correct notation for time, 12- and 24-hour clock and work out time taken for a journey from a timetable;</li><li>• Construct tables for time-series data;</li><li>• Design, complete and use two-way tables for discrete and grouped data;</li><li>• Calculate the total frequency from a frequency table;</li><li>• Read off frequency values from a table;</li><li>• Read off frequency values from a frequency table;</li><li>• Find greatest and least values from a frequency table;</li><li>• Identify the mode from a frequency table;</li><li>• Identify the modal class from a grouped frequency table;</li><li>• Plotting coordinates in first quadrant and read graph scales in multiples;</li><li>• Produce and interpret:<ul style="list-style-type: none"><li>• pictograms;</li><li>• composite bar charts;</li><li>• dual/comparative bar charts for categorical and ungrouped discrete data;</li><li>• bar-line charts;</li><li>• vertical line charts;</li><li>• line graphs;</li><li>• line graphs for time-series data;</li><li>• histograms with equal class intervals;</li><li>• stem and leaf (including back-to-back);</li></ul></li><li>• Calculate total population from a bar chart or table;</li><li>• Find greatest and least values from a bar chart or table;</li><li>• Find the mode from a stem and leaf diagram;</li><li>• Identify the mode from a bar chart;</li><li>• Recognise simple patterns, characteristic and relationships in bar charts and line graphs;</li><li>• Interpret and discuss any data.</li></ul>	<ul style="list-style-type: none"><li>• Construct a frequency table for a continuous data set, deciding on appropriate intervals using inequalities</li><li>• Plan a journey using timetables.</li><li>• Decide the most appropriate chart or table given a data set.</li><li>• State the mode, smallest value or largest value from a stem and leaf diagram</li></ul>

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Misleading graphs, charts or tables can provide an opportunity for students to critically evaluate the way information is presented.</li> <li>• Students should be able to decide what the scales on any axis should be to be able to present information.</li> </ul>	<ul style="list-style-type: none"> <li>• Students struggle to make the link between what the data in a frequency table represents, so for example may state the 'frequency' rather than the interval when asked for the modal group.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Other averages are covered in unit 5, but you may choose to cover them in this unit.</li> <li>• Ensure that students are given the opportunity to draw and complete two-way tables from words.</li> <li>• Ensure that you include a variety of scales, including decimal numbers of millions and thousands, time scales in hours, minutes, seconds.</li> <li>• Misleading graphs are a useful life skill.</li> </ul>	

### 3b. Pie charts (G2, G15, S2, S4)

Estimated 3 hours

Objectives	Possible Success Criteria
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Interpret tables; represent data in tables and charts;</li> <li>• Know which charts to use for different types of data sets;</li> <li>• Draw circles and arcs to a given radius;</li> <li>• Know there are 360 degrees in a full turn, 180 degrees in a half turn, and 90 degrees in a quarter turn;</li> <li>• Measure and draw angles, to the nearest degree; Construct pie charts for categorical data and discrete/continuous numerical data;</li> <li>• Interpret simple pie charts using simple fractions and percentages; <math>\frac{1}{2}</math>, <math>\frac{1}{4}</math> and multiples of 10% sections;</li> <li>• From a pie chart:               <ul style="list-style-type: none"> <li>○ find the mode;</li> <li>○ find the total frequency;</li> </ul> </li> <li>• Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts.</li> </ul>	<ul style="list-style-type: none"> <li>• From a simple pie chart identify the frequency represented by <math>\frac{1}{4}</math> and <math>\frac{1}{2}</math> sections.</li> <li>• From a simple pie chart identify the mode.</li> <li>• Find the angle for one item.</li> </ul>
Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• From inspection of a pie chart, students should be able to identify the fraction of the total represented and know when that total can be calculated and compared with another pie chart.</li> </ul>	<ul style="list-style-type: none"> <li>• Same size sectors for different sized data sets represent the same number rather than the same proportion.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Relate <math>\frac{1}{4}</math>, <math>\frac{1}{2}</math>, etc to percentages.</li> <li>• Practise dividing by 20, 30, 40, 60, etc.</li> <li>• Compare pie charts to identify similarities and differences.</li> <li>• Angles when drawing pie charts should be accurate to 2°.</li> </ul>	

<b>3c. Scatter graphs (S4, S6)</b>		<b>Estimated 4 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Draw scatter graphs;</li> <li>• Interpret points on a scatter graph;</li> <li>• Identify outliers and ignore them on scatter graphs;</li> <li>• Draw the line of best fit on a scatter diagram by eye, and understand what it represents;</li> <li>• Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing;</li> <li>• Distinguish between positive, negative and no correlation using lines of best fit;</li> <li>• Use a line of best fit to predict values of a variable given values of the other variable;</li> <li>• Interpret scatter graphs in terms of the relationship between two variables;</li> <li>• Interpret correlation in terms of the problem;</li> <li>• Understand that correlation does not imply causality;</li> <li>• State how reliable their predictions are, i.e. not reliable if extrapolated.</li> </ul>	<ul style="list-style-type: none"> <li>• Justify an estimate they have made using a line of best fit.</li> <li>• Identify outliers and explain why they may occur.</li> <li>• Given two sets of data in a table, model the relationship and make predictions.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Many real-life situations that give rise to two variables provide opportunities for students to extrapolate and interpret the resulting relationship (if any) between the variables.</li> </ul>	<ul style="list-style-type: none"> <li>• Lines of best fit are often forgotten, but correct answers still obtained by sight.</li> <li>• Interpreting scales of different measurements and confusion between x and y axes when plotting points.</li> </ul>	

Notes	Suggested Activities
<ul style="list-style-type: none"><li>• Students need to be constantly reminded of the importance of drawing a line of best fit.</li><li>• Support with copy and complete statements, e.g. as the ___ increases, the ___ decreases.</li><li>• Statistically the line of best fit should pass through the coordinate representing the mean of the data.</li><li>• Students should label the axes clearly, and use a ruler for all straight lines and a pencil for all drawing.</li><li>• Remind students that the line of best fit does not necessarily go through the origin of the graph.</li></ul>	

## UNIT 4: Fractions and percentages

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### SPECIFICATION REFERENCES

- N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
- N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- N8 calculate exactly with fractions ...
- N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and  $\frac{7}{2}$  or 0.375 and  $\frac{3}{8}$ )
- N12 interpret fractions and percentages as operators
- R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease, and original value problems and simple interest including in financial mathematics
- S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, [tables and line graphs for time series data](#) and know their appropriate use

### PRIOR KNOWLEDGE

- Students should be able to use the four operations of number.
- Students should be able to find common factors.
- Students have a basic understanding of fractions as being 'parts of a whole'.
- Students should be able to define percentage as 'number of parts per hundred'.
- Students should know number complements to 10 and multiplication tables.

### KEYWORDS

Decimal, percentage, inverse, addition, subtraction, multiplication, division, fractions, mixed, improper, recurring, integer, decimal, terminating, percentage, VAT, increase, decrease, multiplier, profit, loss

**4a. Fractions, decimals and percentages (N1, N2, N3, N8, N10, N12, R3, R9 S2)**

**Estimated 7 hours**

Objectives	Possible Success Criteria
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use diagrams to find equivalent fractions or compare fractions;</li> <li>• Write fractions to describe shaded parts of diagrams;</li> <li>• Express a given number as a fraction of another, using very simple numbers, some cancelling, and where the fraction is both <math>&lt; 1</math> and <math>&gt; 1</math>;</li> <li>• Write a fraction in its simplest form and find equivalent fractions;</li> <li>• Order fractions, by using a common denominator;</li> <li>• Compare fractions, use inequality signs, compare unit fractions;</li> <li>• Convert between mixed numbers and improper fractions;</li> <li>• Add and subtract fractions;</li> <li>• Add fractions and write the answer as a mixed number;</li> <li>• Multiply and divide an integer by a fraction;</li> <li>• Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;</li> <li>• Understand and use unit fractions as multiplicative inverses;</li> <li>• Multiply fractions: simplify calculations by cancelling first;</li> <li>• Divide a fraction by a whole number and another fraction;</li> <li>• Recall the fraction-to-decimal conversion and convert fractions to decimals;</li> <li>• Convert a fraction to a decimal to make a calculation easier, e.g. <math>0.25 \times 8 = \frac{1}{4} \times 8</math>, or <math>\frac{3}{8} \times 10 = 0.375 \times 10</math>;</li> <li>• Recognise recurring decimals and convert fractions such as <math>\frac{3}{7}</math>, <math>\frac{1}{3}</math> and <math>\frac{2}{3}</math> into recurring decimals;</li> <li>• Compare and order fractions, decimals and integers, using inequality signs;</li> <li>• Understand that a percentage is a fraction in hundredths;</li> <li>• Express a given number as a percentage of another number;</li> <li>• Convert between fractions, decimals and percentages;</li> <li>• Order fractions, decimals and percentages, including use of inequality signs.</li> </ul>	<ul style="list-style-type: none"> <li>• Express a given number as a fraction of another, including where the fraction <math>&gt; 1</math>.</li> <li>• Simplify <math>\frac{120}{100}</math>.</li> <li>• <math>\frac{3}{5} \times 15</math>, <math>20 \times \frac{3}{4}</math>.</li> <li>• <math>\frac{1}{2}</math> of 36 m, <math>\frac{1}{4}</math> of £20.</li> <li>• Find the size of each category from a pie chart using fractions.</li> <li>• Calculate: <math>\frac{1}{2} \times \frac{6}{7}</math>, <math>\frac{3}{5} \div 3</math>.</li> <li>• Write terminating decimals (up to 3 d.p.) as fractions.</li> <li>• Convert between fractions, decimals and percentages, common ones such as <math>\frac{1}{2}</math>, <math>\frac{1}{10}</math>, <math>\frac{1}{4}</math>, <math>\frac{3}{4}</math> and <math>\frac{n}{10}</math>.</li> <li>• Order integers, decimals and fractions.</li> </ul>

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Questions that involve rates of overtime pay including simple calculations involving fractional (<math>&gt;1</math>, e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.</li> <li>• Working out the number of people/things where the number of people/things in different categories is given as a fraction, decimal or percentage.</li> </ul>	<ul style="list-style-type: none"> <li>• The larger the denominator the larger the fraction.</li> <li>• Incorrect links between fractions and decimals, such as  <math>\frac{1}{5} = 0.15</math>, <math>5\% = 0.5</math>,  <math>4\% = 0.4</math>, etc.</li> <li>• It is not possible to have a percentage greater than 100%.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Emphasise the importance of being able to convert between fractions, decimals and percentages to make calculations easier.</li> <li>• When expressing a given number as a fraction of another, start with very simple numbers <math>&lt; 1</math>, and include some cancelling before fractions using numbers <math>&gt; 1</math>.</li> <li>• Students should be reminded of basic percentages and fraction conversions.</li> <li>• When adding and subtracting fractions, start with same denominator, then where one denominator is a multiple of the other (answers <math>\leq 1</math>), and finally where both denominators have to be changed (answers <math>\leq 1</math>).</li> <li>• Regular revision of fractions is essential.</li> <li>• Demonstrate how to use the fraction button on the calculator.</li> <li>• Use real-life examples where possible.</li> <li>• Use long division to illustrate recurring decimals.</li> </ul>	

<b>4b. Percentages (N12, R9)</b>		<b>Estimated 6 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Express a given number as a percentage of another number;</li> <li>• Find a percentage of a quantity without a calculator: 50%, 25% and multiples of 10% and 5%;</li> <li>• Find a percentage of a quantity or measurement (use measurements they should know from Key Stage 3 only);</li> <li>• Calculate amount of increase/decrease;</li> <li>• Use percentages to solve problems, including comparisons of two quantities using percentages;</li> <li>• Percentages over 100%;</li> <li>• Use percentages in real-life situations, including percentages greater than 100%:</li> <li>• Price after VAT (not price before VAT);</li> <li>• Value of profit or loss;</li> <li>• Simple interest;</li> <li>• Income tax calculations;</li> <li>• Use decimals to find quantities;</li> <li>• Find a percentage of a quantity, including using a multiplier;</li> <li>• Use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;</li> <li>• Understand the multiplicative nature of percentages as operators.</li> </ul>	<ul style="list-style-type: none"> <li>• What is 10%, 15%, 17.5% of £30?</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Sale prices offer an ideal opportunity for solving problems allowing students the opportunity to investigate the most effective way to work out the "sale" price.</li> <li>• Problems that involve consecutive reductions such as: Sale Prices are 10% off the previous day's price. If a jacket is £90 on Monday, what is the price on Wednesday?</li> </ul>	<ul style="list-style-type: none"> <li>• It is not possible to have a percentage greater than 100%.</li> </ul>	

<b>Notes</b>	<b>Suggested Activities</b>
<ul style="list-style-type: none"><li>• When finding a percentage of a quantity or measurement, use only measurements they should know from Key Stage 3.</li><li>• Amounts of money should always be rounded to the nearest penny.</li><li>• Use real-life examples where possible.</li><li>• Emphasise the importance of being able to convert between decimals and percentages and the use of decimal multipliers to make calculations easier.</li></ul>	<ul style="list-style-type: none"><li>•</li></ul>

## UNIT 5: Equations, inequalities and sequences

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### SPECIFICATION REFERENCES

- N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
- N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
- N16 apply and interpret limits of accuracy
- A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors
- A5 understand and use standard mathematical formulae; rearrange formulae to change the subject
- A7 where appropriate, interpret simple expressions as functions with inputs and outputs
- A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
- A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution
- A22 solve linear inequalities in one variable; represent the solution set on a number line
- A23 generate terms of a sequence from either a term-to-term or a position-to-term rule
- A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions; Fibonacci type sequences and simple geometric progressions ( $r^n$  where  $n$  is an integer, and  $r$  is a rational number  $> 0$ )
- A25 deduce expressions to calculate the  $n$ th term of linear sequences.

### PRIOR KNOWLEDGE

Students should be able to use inequality signs between numbers.

Students should be able to use negative numbers with the four operations, recall and use the hierarchy of operations and understand inverse operations.

Students should be able to deal with decimals and negatives on a calculator.

Students should be able to use index laws numerically.

Students should be able to draw a number line.

### KEYWORDS

Arithmetic, geometric, function, sequence,  $n$ th term, derive, quadratic, triangular, cube, square, odd, even, solve, change, subject, inequality, represent, substitute, bracket, expand, linear, equation, balance, accuracy

**5a. Equations and inequalities (N1, N15, N16, A3, A5, A7, A17, A21, A22)****Estimated 9 hours****Objectives**

By the end of the sub-unit, students should be able to:

- Select an expression/equation/formula/identity from a list;
- Write expressions and set up simple equations including forming an equation from a word problem;
- Use function machines;
- Solve simple equations including those:
  - with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
  - which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
  - with one unknown, with integer or fractional coefficients;
- Rearrange simple equations;
- Substitute into a formula, and solve the resulting equation;
- Find an approximate solution to a linear equation using a graph;
- Solve angle or perimeter problems using algebra.
- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve an inequality such as  $-3 < 2x + 1 < 7$  and show the solution set on a number line;
- Solve two inequalities in  $x$ , find the solution sets and compare them to see which value of  $x$  satisfies both;
- Use the correct notation to show inclusive and exclusive inequalities;
- Construct inequalities to represent a set shown on a number line;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Round answers to a given degree of accuracy.

**Possible Success Criteria**

- Solve:  $x + 5 = 12$ ,  $x - 6 = 3$ ,  $\frac{x}{2} = 5$ ,  $2x - 5 = 19$ ,  $2x + 5 = 8x - 7$
- Given expressions for the angles on a line or in a triangle in terms of  $a$ , find the value of  $a$ .
- Given expressions for the sides of a rectangle and the perimeter, form and solve an equation to find missing values.
- Solve  $-3 < 2x + 1$  and show the solution set on a number line.
- State the whole numbers that satisfy a given inequality.
- Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction.

Opportunities for reasoning/problem solving	Common Misconceptions
<p>Problems that:</p> <ul style="list-style-type: none"> <li>could be solved by forming equations such as: Pat and Paul have a combined salary of £800 per week. Pat earns £200 per week more than Paul. How much does Paul earn?</li> <li>involve the application of a formula with conflicting results such as: Pat and Paul are using the formula <math>y = 8n + 4</math>. When <math>n = 2</math>, Pat states that <math>y = 86</math> and Paul states <math>y = 20</math>. Who is correct?</li> </ul>	<ul style="list-style-type: none"> <li>Rules of adding and subtracting negatives.</li> <li>Inverse operations can be misapplied.</li> <li>When solving inequalities, students often state their final answer as a number quantity and either exclude the inequality or change it to <math>=</math>.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>Emphasise good use of notation.</li> <li>Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.</li> <li>Students can leave their answer in fraction form where appropriate.</li> <li>Emphasise the importance of leaving their answer as an inequality (and not change to <math>=</math>).</li> </ul>	

**5b. Sequences****Estimated 5 hours**

(A7, A23, A24, A25)

<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Recognise sequences of odd and even numbers, and other sequences including Fibonacci sequences;</li> <li>• Use function machines to find terms of a sequence;</li> <li>• Write the term-to-term definition of a sequence in words;</li> <li>• Find a specific term in the sequence using position-to-term or term-to-term rules;</li> <li>• Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams;</li> <li>• Recognise such sequences from diagrams and draw the next term in a pattern sequence;</li> <li>• Find the next term in a sequence, including negative values;</li> <li>• Find the <math>n</math>th term               <ul style="list-style-type: none"> <li>• for a pattern sequence;</li> <li>• a linear sequence;</li> <li>• of an arithmetic sequence;</li> </ul> </li> <li>• Use the <math>n</math>th term of an arithmetic sequence to               <ul style="list-style-type: none"> <li>• generate terms;                   <ul style="list-style-type: none"> <li>○ decide if a given number is a term in the sequence, or find the first term over a certain number;</li> </ul> </li> <li>• find the first term greater/less than a certain number;</li> </ul> </li> <li>• Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms;</li> <li>• Continue a quadratic sequence and use the <math>n</math>th term to generate terms;</li> <li>• Distinguish between arithmetic and geometric sequences.</li> </ul>	<ul style="list-style-type: none"> <li>• Given a sequence, 'Which is the 1st term greater than 50?'</li> <li>• What is the amount of money after <math>x</math> months saving the same amount or the height of tree that grows 6 m per year?</li> <li>• What are the next terms in the following sequences?</li> <li>• 1, 3, 9, ...                      100, 50, 25, ...                      2, 4, 8, 16, ...</li> <li>• Write down an expression for the <math>n</math>th term of the arithmetic sequence 2, 5, 8, 11, ...</li> <li>• Is 67 a term in the sequence 4, 7, 10, 13, ...?</li> </ul>
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>
<ul style="list-style-type: none"> <li>• Evaluating statements about whether or not specific numbers or patterns are in a sequence and justifying the reasons.</li> </ul>	

Notes	Suggested Activities
<ul style="list-style-type: none"><li>• Emphasise use of <math>3n</math> meaning <math>3 \times n</math>.</li><li>• Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the <math>n</math>th term.</li><li>• Students are not expected to find the <math>n</math>th term of a quadratic sequence.</li></ul>	

## UNIT 6: Angles, polygons and parallel lines

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### SPECIFICATION REFERENCES

- G1 use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
- G3 apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
- G4 derive and apply the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
- G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including ... the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
- G11 solve geometrical problems on coordinate axes

### PRIOR KNOWLEDGE

Students should be able to use a ruler and protractor.

Students should have an understanding of angles as a measure of turning.

Students should be able to name angles and distinguish between acute, obtuse, reflex and right angles.

Students should recognise reflection symmetry, be able to identify and draw lines of symmetry, and complete diagrams with given number of lines of symmetry.

Students should recognise rotation symmetry and be able to identify orders of rotational symmetry, and complete diagrams with given order of rotational symmetry.

### KEYWORDS

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, rotational symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, triangle, perpendicular, isosceles, scalene, clockwise, anticlockwise, hexagons, heptagons, octagons, decagons, obtuse, acute, reflex, quadrilateral, triangle, regular, irregular, two-dimensional, three-dimensional, measure, line, angle, order, intersecting

**6a. Properties of shapes, parallel lines and angle facts (G1, G3, G4, G6, G11)****Estimated 7 hours****Objectives**

By the end of the sub-unit, students should be able to:

- Estimate sizes of angles;
- Measure angles using a protractor;
- Use geometric language appropriately;
- Use letters to identify points, lines and angles;
- Use two-letter notation for a line and three-letter notation for an angle;
- Describe angles as turns and in degrees and understand clockwise and anticlockwise;
- Know that there are  $360^\circ$  in a full turn,  $180^\circ$  in a half turn and  $90^\circ$  in a quarter turn;
- Identify a line perpendicular to a given line on a diagram and use their properties;
- Identify parallel lines on a diagram and use their properties;
- Find missing angles using properties of corresponding and alternate angles;
- Understand and use the angle properties of parallel lines.
- Recall the properties and definitions of special types of quadrilaterals, including symmetry properties;
- List the properties of each special type of quadrilateral, or identify (name) a given shape;
- Draw sketches of shapes;
- Classify quadrilaterals by their geometric properties and name all quadrilaterals that have a specific property;
- Identify quadrilaterals from everyday usage;
- Given some information about a shape on coordinate axes, complete the shape; Understand and use the angle properties of quadrilaterals;
- Use the fact that angle sum of a quadrilateral is  $360^\circ$ ;
- Recall and use properties of angles at a point, angles at a point on a straight line, right angles, and vertically opposite angles;
- Distinguish between scalene, equilateral, isosceles and right-angled triangles;
- Derive and use the sum of angles in a triangle;
- Find a missing angle in a triangle, using the angle sum of a triangle is  $180^\circ$ ;
- Understand and use the angle properties of triangles, use the symmetry property of isosceles triangle to show that base angles are equal;
- Use the side/angle properties of isosceles and equilateral triangles;
- Understand and use the angle properties of intersecting lines;
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices; Use geometrical language appropriately, give reasons for angle calculations and show step-by-step deduction when solving problems.

**Possible Success Criteria**

- Name all quadrilaterals that have a specific property.
- Use geometric reasoning to answer problems giving detailed reasons.
- Find the size of missing angles at a point or at a point on a straight line.
- Convince me that a parallelogram is a rhombus.

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Multi-step “angle chasing” style problems that involve justifying how students have found a specific angle.</li> <li>• Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.</li> <li>• What is the same, and what is different between families of polygons?</li> </ul>	<ul style="list-style-type: none"> <li>• Pupils may believe, incorrectly, that perpendicular lines have to be horizontal/vertical or all triangles have rotational symmetry of order 3.</li> <li>• Some students will think that all trapezia are isosceles, or a square is only square if ‘horizontal’, or a ‘non-horizontal’ square is called a diamond.</li> <li>• Some students may think that the equal angles in an isosceles triangle are the ‘base angles’.</li> <li>• Incorrectly identifying the ‘base angles’ (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Emphasise that diagrams in examinations are seldom drawn accurately.</li> <li>• Make sure drawings are neat, labelled and accurate.</li> <li>• Give students lots of practice.</li> <li>• Angles should be accurate to within 2°.</li> <li>• Investigate Rangoli patterns.</li> <li>• Use tracing paper to assist with symmetry questions.</li> <li>• Ask students to find their own examples of symmetry in real life.</li> <li>• Emphasise that diagrams in examinations are seldom drawn accurately.</li> <li>• Make sure drawings are neat, labelled and accurate.</li> <li>• Students should have plenty of practice drawing examples to illustrate the properties and encourage them to check their drawings.</li> <li>• Emphasise the need to give geometric reasons when required.</li> </ul>	

<b>6b. Interior and exterior angles of polygons (G1, G3, G5, G11)</b>		<b>Estimated 4 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Recognise and name pentagons, hexagons, heptagons, octagons and decagons;</li> <li>• Understand 'regular' and 'irregular' as applied to polygons;</li> <li>• Use the sum of angles of irregular polygons;</li> <li>• Calculate and use the sums of the interior angles of polygons;</li> <li>• Calculate and use the angles of regular polygons;</li> <li>• Use the sum of the interior angles of an <math>n</math>-sided polygon;</li> <li>• Use the sum of the exterior angles of any polygon is <math>360^\circ</math>;</li> <li>• Use the sum of the interior angle and the exterior angle is <math>180^\circ</math>;</li> <li>• Identify shapes which are congruent (by eye);</li> <li>• Explain why some polygons fit together and others do not;</li> </ul>	<ul style="list-style-type: none"> <li>• Deduce and use the angle sum in any polygon.</li> <li>• Derive the angle properties of regular polygons.</li> <li>• Given the size of its exterior angle, how many sides does the polygon have?</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Problems whereby students have to justify the number of sides that a regular polygon has given an interior or exterior angle.</li> </ul>	<ul style="list-style-type: none"> <li>• Pupils may believe, incorrectly, that all polygons are regular.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Study Escher drawings.</li> <li>• Use examples of tiling patterns with simple shapes to help students investigate if shapes 'fit together'.</li> </ul>		

## UNIT 7: Statistics, sampling and the averages

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### SPECIFICATION REFERENCES

- S1 infer properties of populations or distributions from a sample, while knowing the limitations of sampling
- S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time-series data and know their appropriate use
- S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: ...
- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)

### PRIOR KNOWLEDGE

Students should be able to calculate the midpoint of two numbers.

Students will have drawn the statistical diagrams in unit 3.

Students will have used inequality notation.

### KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate, primary, secondary, interval, midpoint, survey

**7: Statistics, sampling and the averages (S1, S2, S4)****Estimated 7 hours****Objectives**

By the end of the sub-unit, students should be able to:

- Specify the problem and:
  - plan an investigation;
  - decide what data to collect and what statistical analysis is needed;
  - consider fairness;
- Recognise types of data: primary secondary, quantitative and qualitative;
- Identify which primary data they need to collect and in what format, including grouped data;
- Collect data from a variety of suitable primary and secondary sources;
- Understand how sources of data may be biased and explain why a sample may not be representative of a whole population;
- Understand sample and population.
- Calculate the mean, mode, median and range for discrete data;
- Interpret and find a range of averages as follows:
  - median, mean and range from a (discrete) frequency table;
  - range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table;
  - mode and range from a bar chart;
  - median, mode and range from stem and leaf diagrams;
  - mean from a bar chart;
- Understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;
- Recognise the advantages and disadvantages between measures of average.

**Possible Success Criteria**

- Explain why a sample may not be representative of a whole population.
- Carry out a statistical investigation of their own and justify how sources of bias have been eliminated.
- Show me an example of a situation in which biased data would result.
- State the median, mode, mean and range from a small data set.
- Extract the averages from a stem and leaf diagram.
- Estimate the mean from a table.

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• When using a sample of a population to solve contextual problem, students should be able to justify why the sample may not be representative of the whole population.</li> <li>• Students should be able to provide a correct solution as a counter-argument to statements involving the “averages”, e.g. Susan states that the median is 15, she is wrong. Explain why.</li> <li>• Given the mean, median and mode of five positive whole numbers, can you find the numbers?</li> </ul>	<ul style="list-style-type: none"> <li>• The concept of an unbiased sample is difficult for some students to understand.</li> <li>• Often the <math>\Sigma(m \times f)</math> is divided by the number of classes rather than <math>\Sigma f</math> when estimating the mean.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Emphasise the difference between primary and secondary sources and remind students about the different between discrete and continuous data.</li> <li>• Discuss sample size and mention that a census is the whole population (the UK census takes place every 10 years in a year ending with a 1 – the next one is due in 2021).</li> <li>• Specify the problem and planning for data collection is not included in the programme of study but is a prerequisite to understand the context of the topic.</li> <li>• Writing a questionnaire is not part of the new specification, but is a good topic to demonstrate bias and ways to reduce bias in terms of timing, location and question types that can introduce bias.</li> <li>• Encourage students to cross out the midpoints of each group once they have used these numbers to in <math>m \times f</math>. This helps students to avoid summing <math>m</math> instead of <math>f</math>.</li> <li>• Remind students how to find the midpoint of two numbers.</li> <li>• Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.</li> <li>• When comparing the mean and range of two distributions support with ‘copy and complete’ sentences, or suggested wording.</li> </ul>	

## UNIT 8: Perimeter, area and volume

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### SPECIFICATION REFERENCES

- N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- G11 solve geometrical problems on coordinate axes
- G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
- G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
- G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
- G17 ... calculate: perimeters of 2D shapes, including ... composite shapes

### PRIOR KNOWLEDGE

Students should be able to measure lines and recall the names of 2D shapes.

Students should be able to use strategies for multiplying and dividing by powers of 10.

Students should be able to find areas by counting squares and volumes by counting cubes.

Students should be able to interpret scales on a range of measuring instruments.

### KEYWORDS

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, symmetry, vertices, edge, face, units, conversion

**8: Perimeter, area and volume****Estimated 10 hours****Objectives**

- By the end of the sub-unit, students should be able to:
- Indicate given values on a scale, including decimal value;
  - Know that measurements using real numbers depend upon the choice of unit;
  - Convert between units of measure within one system, including time and metric units to metric units of length, area and volume and capacity e.g.  $1\text{ml} = 1\text{cm}^3$ ;
  - Make sensible estimates of a range of measures in everyday settings;
  - Measure shapes to find perimeters and areas using a range of scales;
  - Find the perimeter of
    - rectangles and triangles;
    - parallelograms and trapezia;
    - compound shapes;
  - Recall and use the formulae for the area of a triangle and rectangle;
  - Find the area of a trapezium and recall the formula;
  - Find the area of a parallelogram;
  - Calculate areas and perimeters of compound shapes made from triangles and rectangles;
  - Estimate surface areas by rounding measurements to 1 significant figure;
  - Find the surface area of a prism;
  - Find surface area using rectangles and triangles;
  - Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone;
  - Sketch nets of cuboids and prisms;
  - Recall and use the formula for the volume of a cuboid;
  - Find the volume of a prism, including a triangular prism, cube and cuboid;
  - Calculate volumes of right prisms and shapes made from cubes and cuboids;
  - Estimate volumes etc by rounding measurements to 1 significant figure;

**Possible Success Criteria**

- Find the area/perimeter of a given shape, stating the correct units.
- Justify whether a certain number of small boxes fit inside a larger box.
- Calculate the volume of a triangular prism with correct units.

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Given two 2D that shapes have equal areas, work out all the dimensions of the sides of the shapes.</li> <li>• Problems involving straight-forward and compound shapes in a real-life context should be explored to reinforce the concept of area. For example, the floor plan of a garden linked to the purchase of grass seed.</li> </ul>	<ul style="list-style-type: none"> <li>• Shapes involving missing lengths of sides often result in incorrect answers.</li> <li>• Students often confuse perimeter and area.</li> <li>• Volume often gets confused with surface area.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Use questions that involve different metric measures that need converting.</li> <li>• Measurement is essentially a practical activity: use a range of everyday shapes to bring reality to lessons.</li> <li>• Ensure that students are clear about the difference between perimeter and area.</li> <li>• Practical examples help to clarify the concepts, i.e. floor tiles, skirting board, etc.</li> <li>• Discuss the correct use of units.</li> <li>• Drawings should be done in pencil.</li> <li>• Consider 'how many small boxes fit in a larger box'-type questions.</li> <li>• Practical examples should be used to enable students to understand the difference between perimeter, area and volume.</li> </ul>	

## UNIT 9: Real-life and algebraic linear graphs

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### SPECIFICATION REFERENCES

- N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- A7 where appropriate, interpret simple expressions as functions with inputs and outputs
- A8 work with coordinates in all four quadrants
- A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; ...
- A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically
- A12 Recognise, sketch and interpret graphs of linear functions ...
- A14 plot and interpret ... graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
- R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- R11 use compound units such as speed, ... unit pricing, ...
- R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- G11 solve geometrical problems on coordinate axes
- G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

### PRIOR KNOWLEDGE

Students should be able to plot coordinates and read scales  
Students should be able to substitute into a formula.

### KEYWORDS

Linear, graph, distance, time, coordinate, quadrant, real-life graph, gradient, intercept, function, solution, parallel

9a. Real-life graphs (N13, A7, A8, A9, A10, A14, R1, R11, R14, G11, G14)		Estimated 8 hours
<p><b>Objectives</b></p> <p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use input/output diagrams;</li> <li>• Draw, label and scale axes;</li> <li>• Use axes and coordinates to specify points in all four quadrants in 2D;</li> <li>• Identify points with given coordinates and coordinates of a given point in all four quadrants;</li> <li>• Find the coordinates of points identified by geometrical information in 2D (all four quadrants);</li> <li>• Find the coordinates of the midpoint of a line segment; Read values from straight-line graphs for real-life situations;</li> <li>• Draw straight line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills graphs, fixed charge and cost per unit;</li> <li>• Draw distance–time graphs and velocity–time graphs;</li> <li>• Work out time intervals for graph scales;</li> <li>• Interpret distance–time graphs, and calculate: the speed of individual sections, total distance and total time;</li> <li>• Interpret information presented in a range of linear and non-linear graphs;</li> <li>• Interpret graphs with negative values on axes;</li> <li>• Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs.</li> </ul>	<p><b>Possible Success Criteria</b></p> <ul style="list-style-type: none"> <li>• Interpret a description of a journey into a distance–time or speed–time graph.</li> </ul>	
<p><b>Opportunities for reasoning/problem solving</b></p> <ul style="list-style-type: none"> <li>• Students should be able to decide what the scales on any axis should be to be able to draw a correct graph.</li> <li>• Conversion graphs can be used to provide opportunities for students to justify which distance is further, or whether or not certain items can be purchase in different currencies.</li> </ul>	<p><b>Common Misconceptions</b></p> <ul style="list-style-type: none"> <li>• With distance–time graphs, students struggle to understand that the perpendicular distance from the x-axis represents distance.</li> </ul>	

<b>Notes</b>	<b>Suggested Activities</b>
<ul style="list-style-type: none"><li>• Clear presentation of axes is important.</li><li>• Ensure that you include questions that include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.</li><li>• Careful annotation should be encouraged: it is good practice to get the students to check that they understand the increments on the axes.</li><li>• Use standard units of measurement to draw conversion graphs.</li><li>• Use various measures in distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours.</li></ul>	

**9b. Straight-line graphs (A7, A9, A10, A12, A17)****Estimated 6 hours**

<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use function machines to find coordinates (i.e. given the input <math>x</math>, find the output <math>y</math>);</li> <li>• Plot and draw graphs of <math>y = a</math>, <math>x = a</math>, <math>y = x</math> and <math>y = -x</math>;</li> <li>• Recognise straight-line graphs parallel to the axes;</li> <li>• Recognise that equations of the form <math>y = mx + c</math> correspond to straight-line graphs in the coordinate plane;</li> <li>• Plot and draw graphs of straight lines of the form <math>y = mx + c</math> using a table of values;</li> <li>• Sketch a graph of a linear function, using the gradient and <math>y</math>-intercept;</li> <li>• Identify and interpret gradient from an equation <math>y = mx + c</math>;</li> <li>• Identify parallel lines from their equations;</li> <li>• Plot and draw graphs of straight lines in the form <math>ax + by = c</math>;</li> <li>• Find the equation of a straight line from a graph;</li> <li>• Find the equation of the line through one point with a given gradient;</li> <li>• Find approximate solutions to a linear equation from a graph;</li> <li>• Find the gradient of a straight line from real-life graphs too.</li> </ul>	<ul style="list-style-type: none"> <li>• Plot and draw the graph for <math>y = 2x - 4</math>.</li> <li>• Which of these lines are parallel: <math>y = 2x + 3</math>, <math>y = 5x + 3</math>, <math>y = 2x - 9</math>, <math>2y = 4x - 8</math></li> </ul>
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>
<ul style="list-style-type: none"> <li>• Students should be able to decide what the scales on any axis should in order to draw a correct graph.</li> </ul>	<ul style="list-style-type: none"> <li>• When not given a table of values, students rarely see the relationship between the coordinate axes.</li> </ul>
<b>Notes</b>	<b>Suggested Activities</b>
<ul style="list-style-type: none"> <li>• Emphasise the importance of drawing a table of values when not given one.</li> <li>• Values for a table should be taken from the <math>x</math>-axis.</li> </ul>	

### SPECIFICATION REFERENCES

- R6 express a multiplicative relationship between two quantities as a ratio or a fraction
- G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; ...
- G7 identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)
- G24 describe translations as 2D vectors

### PRIOR KNOWLEDGE

Students should recall basic shapes.

Students should be able to plot points in all four quadrants.

Students should have an understanding of the concept of rotation.

Students should be able to draw and recognise lines parallel to axes and  $y = x$ ,  $y = -x$ .

Students will have encountered the terms clockwise and anticlockwise previously.

### KEYWORDS

Transformation, rotation, reflection, enlargement, translation, single, combination, scale factor, mirror line, centre of rotation, centre of enlargement, column vector, vector, similarity, congruent, angle, direction, coordinate, describe

**10: Transformations (R6, G1, G7, G24)****Estimated 11 hours****Objectives**

By the end of the sub-unit, students should be able to:

- Identify congruent shapes by eye;
- Understand that rotations are specified by a centre, an angle and a direction of rotation;
- Find the centre of rotation, angle and direction of rotation and describe rotations fully using the angle, direction of turn, and centre;
- Rotate and draw the position of a shape after rotation about the origin or any other point including rotations on a coordinate grid;
- Identify correct rotations from a choice of diagrams;
- Understand that translations are specified by a distance and direction using a vector;
- Translate a given shape by a vector;
- Use column vectors to describe and transform 2D shapes using single translations on a coordinate grid;
- Understand that distances and angles are preserved under rotations and translations, so that any figure is congruent under either of these transformations;
- Understand that reflections are specified by a mirror line;
- Identify correct reflections from a choice of diagrams;
- Identify the equation of a line of symmetry;
- Transform 2D shapes using single reflections (including those not on coordinate grids) with vertical, horizontal and diagonal mirror lines;
- Describe reflections on a coordinate grid;
- Scale a shape on a grid (without a centre specified);
- Understand that an enlargement is specified by a centre and a scale factor;
- Enlarge a given shape using (0, 0) as the centre of enlargement, and enlarge shapes with a centre other than (0, 0);
- Find the centre of enlargement by drawing;
- Describe and transform 2D shapes using enlargements by:
  - a positive integer & fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors, or simple fractions;
- Understand that distances and angles are preserved under reflections, so that any figure is congruent under this transformation;
- Understand that similar shapes are enlargements of each other and angles are preserved – define similar in this unit;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements.

**Possible Success Criteria**

- Understand that translations are specified by a distance and direction (using a vector).
- Describe and transform a given shape by either a rotation or a translation.
- Describe and transform a given shape by a reflection.
- Convince me the scale factor is, for example, 2.5.

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.</li> </ul>	<ul style="list-style-type: none"> <li>The directions on a column vector often get mixed up.</li> <li>Student need to understand that the 'units of movement' are those on the axes, and care needs to be taken to check the scale.</li> <li>Correct language must be used: students often use 'turn' rather than 'rotate'.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation they should not include two types.</li> <li>Include rotations with the centre of rotation inside the shape.</li> <li>Use trial and error with tracing paper to find the centre of rotation.</li> <li>It is essential that the students check the increments on the coordinate grid when translating shapes.</li> <li>Students may need reminding about how to find the equations of straight lines, including those parallel to the axes.</li> <li>When reflecting shapes, the students must include mirror lines on or through original shapes.</li> <li>As an extension, consider reflections with the mirror line through the shape and enlargements with the centre of enlargement inside the shape.</li> <li>NB enlargement using negative scale factors is not included.</li> </ul>	

### SPECIFICATION REFERENCES

- N11 identify and work with fractions in ratio problems
- N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- R4 use ratio notation, including reduction to simplest form
- R5 divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
- R6 express a multiplicative relationship between two quantities as a ratio or a fraction
- R7 understand and use proportion as equality of ratios
- R8 relate ratios to fractions and to linear functions
- R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations
- R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
- R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

### PRIOR KNOWLEDGE

Students should know the four operations of number.

Students should have a basic understanding of fractions as being 'parts of a whole'.

### KEYWORDS

Ratio, proportion, share, parts, fraction, function, direct proportion, inverse proportion, graphical, linear, compare

<b>11a. Ratio (N11, N13, R1, R4, R5, R6, R8, R12)</b>		<b>Estimated 4 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand and express the division of a quantity into a of number parts as a ratio;</li> <li>• Write ratios in their simplest form;</li> <li>• Write/interpret a ratio to describe a situation;</li> <li>• Share a quantity in a given ratio including three-part ratios;</li> <li>• Solve a ratio problem in context:               <ul style="list-style-type: none"> <li>• use a ratio to find one quantity when the other is known;</li> <li>• use a ratio to compare a scale model to a real-life object;</li> <li>• use a ratio to convert between measures and currencies;</li> </ul> </li> <li>• problems involving mixing, e.g. paint colours, cement and drawn conclusions;</li> <li>• Compare ratios;</li> <li>• Write ratios in form <math>1 : m</math> or <math>m : 1</math>;</li> <li>• Write a ratio as a fraction;</li> <li>• Write a ratio as a linear function;</li> <li>• Write lengths, areas and volumes of two shapes as ratios in simplest form;</li> <li>• Express a multiplicative relationship between two quantities as a ratio or a fraction.</li> </ul>	<ul style="list-style-type: none"> <li>• Write a ratio to describe a situation such as 1 blue for every 2 red, or 3 adults for every 10 children.</li> <li>• Recognise that two paints mixed red to yellow <math>5 : 4</math> and <math>20 : 16</math> are the same colour.</li> <li>• Express the statement 'There are twice as many girls as boys' as the ratio <math>2 : 1</math> or the linear function <math>y = 2x</math>, where <math>x</math> is the number of boys and <math>y</math> is the number of girls.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Problems involving sharing in a ratio that include percentages rather than specific numbers, such as: In a youth club the ratio of the number of boys to the number of girls is <math>3 : 2</math>. 30% of the boys are under the age of 14, and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?</li> </ul>	<ul style="list-style-type: none"> <li>• Students find three-part ratios difficult.</li> <li>• Using a ratio to find one quantity when the other is known often results in students 'sharing' the known amount.</li> </ul>	

Notes	Suggested Activities
<ul style="list-style-type: none"><li>• Emphasise the importance of reading the question carefully.</li><li>• Include ratios with decimals 0.2 : 1.</li><li>• Converting imperial units to imperial units aren't specifically in the programme of study, but still useful and provide a good context for multiplicative reasoning.</li><li>• It is also useful generally for students to know rough metric equivalents of commonly used imperial measures, such as pounds, feet, miles and pints.</li></ul>	

<b>11b. Proportion (R1, R5, R7, R10, R14)</b>		<b>Estimated 5 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand and use proportion as equality of ratios;</li> <li>• Solve word problems involving direct and indirect proportion;</li> <li>• Work out which product is the better buy;</li> <li>• Scale up recipes;</li> <li>• Convert between currencies;</li> <li>• Find amounts for 3 people when amount for 1 given;</li> <li>• Solve proportion problems using the unitary method;</li> <li>• Recognise when values are in direct proportion by reference to the graph form;</li> <li>• Understand inverse proportion: as <math>x</math> increases, <math>y</math> decreases (inverse graphs done in later unit);</li> <li>• Recognise when values are in direct proportion by reference to the graph form;</li> <li>• Understand direct proportion ---&gt; relationship <math>y = kx</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.</li> <li>• If it takes 2 builders 10 days to build a wall, how long will it take 3 builders?</li> <li>• Scale up recipes and decide if there is enough of each ingredient.</li> <li>• Given two sets of data in a table, are they in direct proportion?</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Problems in context, such as scaling a recipe, or diluting lemonade or chemical solutions, will show how proportional reasoning is used in real-life contexts.</li> </ul>		
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Find out/prove whether two variables are in direct proportion by plotting the graph and using it as a model to read off other values.</li> <li>• Possible link with scatter graphs.</li> </ul>		

## UNIT 12: Right-angled triangles: Pythagoras and trigonometry

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### SPECIFICATION REFERENCES

- N7 calculate with roots, and with integer indices
- N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); ...
- A4 simplify and manipulate algebraic expressions (including those involving surds) by: collecting like terms, multiplying a single term over a bracket, ...
- G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
- G20 know the formulae for: Pythagoras' Theorem  $a^2 + b^2 = c^2$  and the trigonometric ratios, sine, cosine and tan; apply them to find angles and lengths in right-angled triangles in two dimensional figures
- G21 know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$ ; know the exact value of  $\tan \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$

### PRIOR KNOWLEDGE

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trigonometric formulae.

Students should recall basic angle facts.

Students should understand when to leave an answer in surd form.

Students can plot coordinates in all four quadrants and draw axes.

### KEYWORDS

Triangle, right angle, angle, Pythagoras' Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, length, accuracy

**12: Right-angled triangles: Pythagoras and trigonometry (N7, N15, A4, G6, G20, G21)**

**Estimated 5 hours**

<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand, recall and use Pythagoras' Theorem in 2D, including leaving answers in surd form and being able to justify if a triangle is right-angled or not;</li> <li>• Calculate the length of the hypotenuse and of a shorter side in a right-angled triangle, including decimal lengths and a range of units;</li> <li>• Apply Pythagoras' Theorem with a triangle drawn on a coordinate grid;</li> <li>• Calculate the length of a line segment AB given pairs of points;</li> <li>• Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;</li> <li>• Use the trigonometric ratios to solve 2D problems including angles of elevation and depression;</li> <li>• Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question;</li> <li>• Know the exact values of <math>\sin \theta</math> and <math>\cos \theta</math> for <math>\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ</math> and <math>90^\circ</math>; know the exact value of <math>\tan \theta</math> for <math>\theta = 0^\circ, 30^\circ, 45^\circ</math> and <math>60^\circ</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• Does 2, 3, 6 give a right angled triangle?</li> <li>• Justify when to use Pythagoras' Theorem and when to use trigonometry.</li> </ul>
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>
<ul style="list-style-type: none"> <li>• Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.</li> <li>• In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed <math>65^\circ</math>. What is the greatest height it can reach?</li> </ul>	<ul style="list-style-type: none"> <li>• Answers may be displayed on a calculator in surd form.</li> <li>• Students forget to square root their final answer or round their answer prematurely.</li> </ul>

Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Students may need reminding about surds.</li> <li>• Drawing the squares on the 3 sides will help to illustrate the theorem.</li> <li>• Include examples with triangles drawn in all four quadrants.</li> <li>• Scale drawings are not acceptable.</li> <li>• Calculators need to be in degree mode.</li> <li>• To find in right-angled triangles the exact values of <math>\sin \theta</math> and <math>\cos \theta</math> for <math>\theta = 0^\circ</math>, <math>30^\circ</math>, <math>45^\circ</math>, <math>60^\circ</math> and <math>90^\circ</math>, use triangles with angles of <math>30^\circ</math>, <math>45^\circ</math> and <math>60^\circ</math>.</li> <li>• Use a suitable mnemonic to remember SOHCAHTOA.</li> <li>• Use Pythagoras' Theorem and trigonometry together.</li> </ul>	

## UNIT 13: Probability

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### SPECIFICATION REFERENCES

- N5 apply systematic listing strategies
- P1 record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
- P2 apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
- P3 relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale
- P4 apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- P5 understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- P6 enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
- P7 construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
- P8 calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

### PRIOR KNOWLEDGE

Students should know how to add and multiply fractions and decimals.

Students should have experience of expressing one number as a fraction of another number.

### KEYWORDS

Probability, dependent, independent, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, fairness, experimental

**13: Probability (P1 to P8)****Estimated 12 hours****Objectives**

By the end of the sub-unit, students should be able to:

- Distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur;
- Mark events and/or probabilities on a probability scale of 0 to 1;
- Write probabilities in words or fractions, decimals and percentages;
- Find the probability of an event happening using theoretical probability;
- Use theoretical models to include outcomes using dice, spinners, coins;
- List all outcomes for single events systematically;
- Work out probabilities from frequency tables, frequency trees, and two way tables;
- Record outcomes of probability experiments in tables;
- Add simple probabilities;
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1;
- Using  $1 - p$  as the probability of an event not occurring where  $p$  is the probability of the event occurring;
- Find a missing probability from a list or table including algebraic terms;
- Find the probability of an event happening using relative frequency;
- Estimate the number of times an event will occur, given the probability and the number of trials – for both experimental and theoretical probabilities;
- List all outcomes for combined events systematically;
- Use and draw sample space diagrams;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes;
- Find the probability of successive events, such as several throws of a single dice;
- Use tree diagrams to calculate the probability of two independent events;
- Use tree diagrams to calculate the probability of two dependent events.

**Possible Success Criteria**

- Mark events on a probability scale and use the language of probability.
- If the probability of outcomes are  $x$ ,  $2x$ ,  $4x$ ,  $3x$  calculate  $x$ .
- Calculate the probability of an event from a two-way table or frequency table.
- Decide if a coin, spinner or game is fair.
- Understand the use of the 0–1 scale to measure probability.
- List all the outcomes for an experiment.
- Know and apply the fact that the sum of probabilities for all outcomes is 1.
- Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Lotteries provides a real life link to probability. Work out the probabilities of winning on different lotteries.</li> <li>• Students should be given the opportunity to justify the probability of events happening or not happening.</li> </ul>	<ul style="list-style-type: none"> <li>• Not using fractions or decimals when working with probability trees.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Use this as an opportunity for practical work.</li> <li>• Probabilities written in fraction form should be cancelled to their simplest form.</li> <li>• Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.</li> <li>• Encourage students to work 'across' the branches working out the probability of each successive event. The probability of the combinations of outcomes should = 1.</li> <li>• Emphasise that were an experiment repeated it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.</li> <li>• Probabilities written in fraction form should be cancelled to their simplest form.</li> </ul>	

## UNIT 14: Multiplicative reasoning: more percentages, rates of change, compound measures

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### SPECIFICATION REFERENCES

- N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- R9 ... express one quantity as a percentage of another; ... solve problems involving percentage change, ... and original value problems ... including in financial mathematics
- R11 use compound units such as speed, rates of pay, unit pricing, density and pressure
- R13 understand that  $X$  is inversely proportional to  $Y$  is equivalent to  $X$  is proportional to  $\frac{1}{Y}$ ; interpret equations that describe direct and inverse proportion
- R16 set up, solve and interpret the answers in growth and decay problems, including compound interest
- G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc)

### PRIOR KNOWLEDGE

Students should be able to interpret scales on a range of measuring instruments.  
Students should be able to find a percentage of an amount and relate percentages to decimals.  
Students should be able to rearrange equations and use these to solve problems.  
Students should know speed = distance/time, density = mass/volume.

### KEYWORDS

Ratio, proportion, best value, proportional change, compound measure, density, mass, volume, speed, distance, time, density, mass, volume, pressure, acceleration, velocity, inverse, direct

**14: Multiplicative reasoning: more percentages, rates of change, compound measures (N13, R1, R9, R11, R13, R16, G14)**

**Estimated 7 hours**

Objectives	Possible Success Criteria
<p>By the end of the unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand and use compound measures:               <ul style="list-style-type: none"> <li>○ density;</li> <li>○ pressure;</li> <li>○ speed;</li> </ul> </li> <li>• convert between metric speed measures;</li> <li>• read values in km/h and mph from a speedometer;</li> <li>• calculate average speed, distance, time – in miles per hour as well as metric measures;</li> <li>• use kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question);</li> <li>• change d/t in m/s to a formula in km/h, i.e. <math>d/t \times (60 \times 60)/1000</math> – with support;</li> <li>• Express a given number as a percentage of another number in more complex situations;</li> <li>• Calculate percentage profit or loss;</li> <li>• Make calculations involving repeated percentage change, not using the formula;</li> <li>• Find the original amount given the final amount after a percentage increase or decrease;</li> <li>• Use compound interest;</li> <li>• Use a variety of measures in ratio and proportion problems:</li> <li>• currency conversion;</li> <li>• rates of pay;</li> <li>• best value;</li> <li>• Set up, solve and interpret the answers in growth and decay problems;</li> <li>• Understand that <math>X</math> is inversely proportional to <math>Y</math> is equivalent to <math>X</math> is proportional to <math>\frac{1}{Y}</math></li> <li>• Interpret equations that describe direct and inverse proportion.</li> </ul>	<ul style="list-style-type: none"> <li>• Know that measurements using real numbers depend upon the choice of unit, with speedometers and rates of change.</li> <li>• Change m/s to km/h.</li> <li>• Understand direct proportion as: as <math>x</math> increase, <math>y</math> increases.</li> <li>• Understand inverse proportion as: as <math>x</math> increases, <math>y</math> decreases.</li> </ul>

Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Speed/distance type problems that involve students justifying their reasons why one vehicle is faster than another.</li> <li>• Calculations involving value for money are a good reasoning opportunity that utilise different skills.</li> <li>• Working out best value of items using different currencies given an exchange rate.</li> </ul>	<ul style="list-style-type: none"> <li>• Some students may think that compound interest and simple interest are the same method of calculating interest.</li> <li>• Incomplete methods when using multipliers, i.e. reduce £80 by 15% = <math>80 \times 0.15</math>.</li> </ul>
Notes	Suggested Activities
<ul style="list-style-type: none"> <li>• Encourage students to use a single multiplier.</li> <li>• Include simple fractional percentages of amounts with compound interest and encourage use of single multipliers.</li> <li>• Amounts of money should be rounded to the nearest penny, but emphasise the importance of not rounding until the end of the calculation if doing in stages.</li> <li>• Use a formula triangle to help students see the relationship for compound measures – this will help them evaluate which inverse operations to use.</li> <li>• Help students to recognise the problem they are trying to solve by the unit measurement given, e.g. km/h is a unit of speed as it is speed divided by a time.</li> </ul>	

## UNIT 15: Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

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### SPECIFICATION REFERENCES

- R2 use scale factors, scale diagrams and maps
- G1 use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description;
- G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
- G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
- G13 construct and interpret plans and elevations of 3D shapes
- G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

### PRIOR KNOWLEDGE

Students should be able to measure and draw lines.

### KEYWORDS

Construct, circle, arc, sector, face, edge, vertex, two-dimensional, three-dimensional, solid, elevations, congruent, angles, regular, irregular, bearing, degree, bisect, perpendicular, loci, map, scale, plan, region

<b>15a. Plans and elevations (G1, G2, G9, G12, G13, G15)</b>		<b>Estimated 5 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand clockwise and anticlockwise;</li> <li>• Draw circles and arcs to a given radius or given the diameter;</li> <li>• Measure and draw lines, to the nearest mm;</li> <li>• Measure and draw angles, to the nearest degree;</li> <li>• Know and use compass directions;</li> <li>• Draw sketches of 3D solids;</li> <li>• Know the terms face, edge and vertex;</li> <li>• Identify and sketch planes of symmetry of 3D solids;</li> <li>• Use isometric grids to draw 2D representations of 3D solids;</li> <li>• Make accurate drawings of triangles and other 2D shapes using a ruler and a protractor;</li> <li>• Construct diagrams of everyday 2D situations involving rectangles, triangles, perpendicular and parallel lines;</li> <li>• Understand and draw front and side elevations and plans of shapes made from simple solids;</li> <li>• Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.</li> </ul>	<ul style="list-style-type: none"> <li>• Be able to estimate the size of given angles.</li> <li>• Convert fluently between metric units of length.</li> <li>• Use bearings in a real-life context to describe the bearing between two towns on a map.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Interpreting scale drawings and maps involving lengths that need to be measured (rather than given in the problem).</li> </ul>	<ul style="list-style-type: none"> <li>• Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as <math>60^\circ</math> rather than as <math>120^\circ</math>.</li> <li>• Often 5 sides only are drawn for a cuboid.</li> </ul>	

<b>Notes</b>	<b>Suggested Activities</b>
<ul style="list-style-type: none"><li>• This is a very practical topic, and provides opportunities for some hands-on activities.</li><li>• Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.</li></ul>	

<b>15b. Constructions, loci and bearings (R2, G2, G5, G15)</b> <span style="float: right;"><b>Estimated 7 hours</b></span>	
<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand congruence, as two shapes that are the same size and shape;</li> <li>• Visually identify shapes which are congruent;</li> <li>• Use straight edge and a pair of compasses to do standard constructions:</li> <li>• understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not;</li> <li>• construct the perpendicular bisector of a given line;</li> <li>• construct the perpendicular from a point to a line;</li> <li>• construct the bisector of a given angle;</li> <li>• construct angles of <math>90^\circ</math>, <math>45^\circ</math>;</li> <li>• Draw and construct diagrams from given instructions, including the following: <ul style="list-style-type: none"> <li>• a region bounded by a circle and an intersecting line;</li> <li>• a given distance from a point and a given distance from a line;</li> <li>• equal distances from two points or two line segments;</li> <li>• regions may be defined by 'nearer to' or 'greater than';</li> </ul> </li> <li>• Find and describe regions satisfying a combination of loci;</li> <li>• Use constructions to solve loci problems (2D only);</li> <li>• Use and interpret maps and scale drawings;</li> <li>• Estimate lengths using a scale diagram;</li> <li>• Make an accurate scale drawing from a diagram;</li> <li>• Use three-figure bearings to specify direction;</li> <li>• Mark on a diagram the position of point <math>B</math> given its bearing from point <math>A</math>;</li> <li>• Give a bearing between the points on a map or scaled plan;</li> <li>• Given the bearing of a point <math>A</math> from point <math>B</math>, work out the bearing of <math>B</math> from <math>A</math>;</li> <li>• Use accurate drawing to solve bearings problems;</li> <li>• Solve locus problems including bearings.</li> </ul>	<ul style="list-style-type: none"> <li>• Sketch the locus of point on a vertex of a rotating shape as it moves along a line, i.e. a point on the circumference or at the centre of a wheel.</li> </ul>
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>
<ul style="list-style-type: none"> <li>• Link problems with other areas of mathematics, such as the trigonometric ratios and Pythagoras' Theorem.</li> </ul>	<ul style="list-style-type: none"> <li>• Correct use of a protractor may be an issue.</li> </ul>

<b>Notes</b>	<b>Suggested Activities</b>
<ul style="list-style-type: none"><li>• Drawings should be done in pencil.</li><li>• Relate loci problems to real-life scenarios, including mobile phone masts and coverage.</li><li>• Construction lines should not be erased.</li></ul>	

## UNIT 16: Algebra: quadratic equations and graphs

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### SPECIFICATION REFERENCES

- A4 simplify and manipulate algebraic expressions by: ... expanding products of two binomials; factorising return to overview expressions of the form  $x^2 + bx + c$ , including the difference of two squares; ...
- A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically
- A12 recognise, sketch and interpret graphs of ... quadratic functions; ...
- A18 solve quadratic equations algebraically by factorising; find approximate solutions using a graph

### PRIOR KNOWLEDGE

Students should be able to square negative numbers.

Students should be able to substitute into formulae.

Students should be able to plot points on a coordinate grid.

Students should be able to expand single brackets and collect 'like' terms.

### KEYWORDS

Quadratic, function, solve, expand, factorise, simplify, expression, graph, curve, factor, coefficient, bracket

<b>16a. Quadratic equations: expanding and factorising (A4, A11, A18)</b>		<b>Estimated 5 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Define a 'quadratic' expression;</li> <li>• Multiply together two algebraic expressions with brackets;</li> <li>• Square a linear expression, e.g. <math>(x + 1)^2</math>;</li> <li>• Factorise quadratic expressions of the form <math>x^2 + bx + c</math>;</li> <li>• Factorise a quadratic expression <math>x^2 - a^2</math> using the difference of two squares;</li> <li>• Solve quadratic equations by factorising;</li> <li>• Find the roots of a quadratic function algebraically.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve <math>3x^2 + 4 = 100</math>.</li> <li>• Expand <math>(x + 2)(x + 6)</math>.</li> <li>• Factorise <math>x^2 + 7x + 10</math>.</li> <li>• Solve <math>x^2 + 7x + 10 = 0</math>.</li> <li>• Solve <math>(x - 3)(x + 4) = 0</math>.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Visual proof of the difference of two squares.</li> </ul>	<ul style="list-style-type: none"> <li>• <math>x</math> terms can sometimes be 'collected' with <math>x^2</math>.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• This unit can be extended by including quadratics where <math>a \neq 1</math>.</li> <li>• Emphasise the fact that <math>x^2</math> and <math>x</math> are different 'types' of term – illustrate this with numbers.</li> </ul>		

<b>16b. Quadratic equations: graphs (A11, A12, A18)</b>		<b>Estimated 4 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;</li> <li>• Identify the line of symmetry of a quadratic graph;</li> <li>• Find approximate solutions to quadratic equations using a graph;</li> <li>• Interpret graphs of quadratic functions from real-life problems;</li> <li>• Identify and interpret roots, intercepts and turning points of quadratic graphs.</li> </ul>	<ul style="list-style-type: none"> <li>• Recognise a quadratic graph from its shape.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Matching graphs with their respective functions.</li> </ul>	<ul style="list-style-type: none"> <li>• Squaring negative numbers can be a problem.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• The graphs should be drawn freehand and in pencil, joining points using a smooth curve.</li> <li>• Encourage efficient use of the calculator.</li> <li>• Extension work can be through plotting cubic and reciprocal graphs, solving simultaneous equations graphically.</li> </ul>	<ul style="list-style-type: none"> <li>•</li> </ul>	

## UNIT 17: Perimeter, area and volume 2: circles, cylinders, cones and spheres

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### SPECIFICATION REFERENCES

- N8 calculate exactly with multiples of  $\pi$
- N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
- G17 know the formulae: circumference of a circle =  $2\pi r = \pi d$ , area of a circle =  $\pi r^2$ ; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids
- G18 calculate arc lengths, angles and areas of sectors of circles

### PRIOR KNOWLEDGE

Students should know the formula for calculating the area of a rectangle.  
Students should know how to use the four operations on a calculator.

### KEYWORDS

Area, perimeter, formula, length, width, measurement, volume, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, sphere, cone, hemisphere, segment, accuracy, surface area

**17: Perimeter, area and volume 2: circles, cylinders, cones and spheres (N8, N14, G9, G16 to G18)**

**Estimated 6 hours**

<b>Objectives</b>	<b>Possible Success Criteria</b>
<p>By the end of the unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Recall the definition of a circle and identify, name and draw parts of a circle including tangent, chord and segment;</li> <li>• Recall and use formulae for the circumference of a circle and the area enclosed by a circle circumference of a circle = <math>2\pi r = \pi d</math>, area of a circle = <math>\pi r^2</math>;</li> <li>• Use <math>\pi \approx 3.142</math> or use the <math>\pi</math> button on a calculator;</li> <li>• Give an answer to a question involving the circumference or area of a circle in terms of <math>\pi</math>;</li> <li>• Find radius or diameter, given area or perimeter of a circles;</li> <li>• Find the perimeters and areas of semicircles and quarter-circles;</li> <li>• Calculate perimeters and areas of composite shapes made from circles and parts of circles;</li> <li>• Calculate arc lengths, angles and areas of sectors of circles;</li> <li>• Find the surface area and volume of a cylinder;</li> <li>• Find the surface area and volume of spheres, pyramids, cones and composite solids;</li> <li>• Round answers to a given degree of accuracy.</li> </ul>	<ul style="list-style-type: none"> <li>• Recall terms related to a circle.</li> <li>• Understand that answers in terms of pi are more accurate.</li> </ul>
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>
<ul style="list-style-type: none"> <li>• Calculate the radius/diameter given the area/circumference type questions could be explored, including questions that require evaluation of statements, such as Andy states "Diameter = <math>2 \times</math> Radius" and Bob states "Radius = <math>2 \times</math> Diameter". Who is correct?</li> </ul>	<ul style="list-style-type: none"> <li>• Diameter and radius are often confused and recollection which formula to use for area and circumference of circles is often poor.</li> </ul>

Notes	Suggested Activities
<ul style="list-style-type: none"><li>• Emphasise the need to learn the circle formula: 'Cherry Pie's Delicious' and 'Apple Pies are too' are good ways to remember them.</li><li>• Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone, will be given on the formulae sheet in the examination.</li><li>• Ensure that students know it is more accurate to leave answers in terms of <math>\pi</math> but only when asked to do so.</li></ul>	

## UNIT 18: More fractions, reciprocals, standard form, zero and negative indices

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### SPECIFICATION REFERENCES

- N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- N7 calculate with roots, and with integer indices
- N9 calculate with and interpret standard form  $A \times 10^n$ , where  $1 \leq A < 10$  and  $n$  is an integer.

### PRIOR KNOWLEDGE

Students should know how to do the four operations with fractions.

Students should be able to write powers of 10 in index form and recognise and recall powers of 10, i.e.  $10^2 = 100$ .

Students should recall the index laws.

### KEYWORDS

Add, subtract, multiply, divide, mixed, improper, fraction, decimal, indices, standard form, power, reciprocal, index

<b>18a. Fractions (N2, N3)</b>		<b>Estimated 5 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Add and subtract mixed number fractions;</li> <li>• Multiply mixed number fractions;</li> <li>• Divide mixed numbers by whole numbers and vice versa;</li> <li>• Find the reciprocal of an integer, decimal or fraction;</li> <li>• Understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is not defined).</li> </ul>	<ul style="list-style-type: none"> <li>• What is the reciprocal of <math>4, \frac{1}{2}, -2, -\frac{1}{2}</math>?</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Students should be able to justify when fractions are equal and provide correct answers as a counter-argument.</li> <li>• Links with other areas of mathematics should be used where appropriate to embed the notion that fractions are not just used in isolation, e.g. use <math>6 \frac{1}{2}</math> cm instead of 6.5 cm.</li> </ul>	<ul style="list-style-type: none"> <li>• The larger the denominator the larger the fraction.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Regular revision of fractions is essential.</li> <li>• Demonstrate how to use the fraction button on the calculator.</li> <li>• Use real-life examples where possible.</li> </ul>		

<b>18b. Indices and standard form (N7, N9)</b>		<b>Estimated 5 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractions and powers of a power;</li> <li>• Use numbers raised to the power zero, including the zero power of 10;</li> <li>• Convert large and small numbers into standard form and vice versa;</li> <li>• Add, subtract, multiply and divide numbers in standard form;</li> <li>• Interpret a calculator display using standard form and know how to enter numbers in standard form.</li> </ul>	<ul style="list-style-type: none"> <li>• Write 51 080 in standard form.</li> <li>• Write <math>3.74 \times 10^{-6}</math> as an ordinary number.</li> <li>• What is <math>9^0</math>?</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Link with other areas of mathematics, such as compound measures, by using speed of light in standard form.</li> </ul>	<ul style="list-style-type: none"> <li>• Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.</li> <li>• When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Negative fractional indices are not included at Foundation tier, but you may wish to extend the work to include these.</li> <li>• Standard form is used in science and there are lots of cross curricular opportunities.</li> <li>• Students need to be provided with plenty of practice in using standard form with calculators.</li> </ul>		

## UNIT 19: Congruence, similarity and vectors

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### SPECIFICATION REFERENCES

- R6 express a multiplicative relationship between two quantities as a ratio or a fraction
- R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
- G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- G7 identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)
- G19 apply the concepts of congruence and similarity, including the relationships between lengths in similar figures
- G24 describe translations as 2D vectors
- G25 apply addition and subtraction of vectors, multiplication by vectors by a scalar, and diagrammatic and column representations of vectors

### PRIOR KNOWLEDGE

Students will have used column vectors when dealing with translations.

Students can recall and apply Pythagoras' Theorem on a coordinate grid.

Students should be able to recognise and enlarge shapes and calculate scale factors.

Students know how to calculate area and volume in various metric measures.

Students should be able to measure lines and angles and using compasses, ruler and protractor, and construct standard constructions.

### KEYWORDS

Vector, direction, magnitude, scalar, multiple, parallel, collinear, ratio, column vector, congruence, side, angle, compass, construction, shape, volume, length, area, volume, scale factor, enlargement, similar, perimeter,

<b>19a. Similarity and congruence in 2D (R6, R12, G5, G7, G19)</b>		<b>Estimated 7 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS);</li> <li>• Solve angle problems involving congruence;</li> <li>• Identify shapes which are similar; including all circles or all regular polygons with equal number of sides;</li> <li>• Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity;</li> <li>• Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides;</li> <li>• Understand the effect of enlargement on perimeter of shapes;</li> <li>• Solve problems to find missing lengths in similar shapes;</li> <li>• Know that scale diagrams, including bearings and maps are 'similar' to the real-life examples.</li> </ul>	<ul style="list-style-type: none"> <li>• Understand similarity as one shape being an enlargement of the other.</li> <li>• Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size.</li> <li>• Use AB notation for describing lengths and <math>\angle ABC</math> notation for describing angles.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Using scale diagrams, including bearings and maps, provides a rich source of real-life examples and links to other areas of mathematics.</li> </ul>	<ul style="list-style-type: none"> <li>• Students may incorrectly believe that all polygons are regular or that all triangles have a rotational symmetry of order 3.</li> <li>• Often students think that when a shape is enlarged the angles also get bigger.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Use simple scale factors that are easily calculated mentally to introduce similar shapes.</li> <li>• Reinforce the fact that the sizes of angles are maintained when a shape is enlarged.</li> <li>• Make links between similarity and trigonometric ratios.</li> </ul>		

<b>19b. Vector (G24, G25)</b>		<b>Estimated 7 hours</b>
<b>Objectives</b>	<b>Possible Success Criteria</b>	
<p>By the end of the sub-unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Understand and use column notation in relation to vectors;</li> <li>• Be able to represent information graphically given column vectors;</li> <li>• Identify two column vectors which are parallel;</li> <li>• Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector.</li> </ul>	<ul style="list-style-type: none"> <li>• Know that if one vector is a multiple of the other, they are parallel.</li> <li>• Add and subtract vectors using column vectors.</li> </ul>	
<b>Opportunities for reasoning/problem solving</b>	<b>Common Misconceptions</b>	
<ul style="list-style-type: none"> <li>• Investigations involving vectors around 2D shapes such as a square can be extended to include considering the area enclosed in the same shapes.</li> </ul>	<ul style="list-style-type: none"> <li>• Students find it difficult to understand that two vectors can be parallel and equal as they can be in different locations in the plane.</li> </ul>	
<b>Notes</b>	<b>Suggested Activities</b>	
<ul style="list-style-type: none"> <li>• Students find manipulation of column vectors relatively easy compared to the pictorial and algebraic manipulation methods – encourage them to draw any vectors that they calculate on the picture.</li> </ul>		

## UNIT 20: Rearranging equations, graphs of cubic and reciprocal functions and simultaneous equations

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### SPECIFICATION REFERENCES

- N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
- A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors
- A5 understand and use standard mathematical formulae; rearrange formulae to change the subject
- A6 ... argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- A9 ... use the form  $y = mx + c$  to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
- A12 recognise, sketch and interpret graphs of ... the reciprocal function  $y = \frac{1}{x}$  with  $x \neq 0$
- A14 plot and interpret ... reciprocal graphs ...
- A19 solve two simultaneous equations in two variables (linear/linear) algebraically; find approximate solutions using a graph
- A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.
- A22 solve linear inequalities in one or two variable(s) represent the solution set on a number line
- R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations
- R14 ... recognise and interpret graphs that illustrate direct and inverse proportion

### PRIOR KNOWLEDGE

Students should be able to draw linear graphs.

Students should be able to plot coordinates and sketch simple functions with a table of values.

Students should be able to substitute into and solve equations.

Students should have experience of using formulae.

Students should recall and use the hierarchy of operations and use of inequality symbols.

### KEYWORDS

Reciprocal, linear, gradient, functions, direct, indirect, estimate, cubic, subject, rearrange, simultaneous, substitution, elimination, proof

**20: Rearranging equations, graphs of cubic and reciprocal functions and simultaneous equations (N1, A3, A5, A6, A9, A12, A14, A19, A21, A22, R10, R14)**

**Estimated 5 hours**

Objectives	Possible Success Criteria
<p>By the end of the unit, students should be able to:</p> <ul style="list-style-type: none"> <li>• Know the difference between an equation and an identity and use and understand the <math>\neq</math> symbol;</li> <li>• Change the subject of a formula involving the use of square roots and squares;</li> <li>• Answer 'show that' questions using consecutive integers <math>(n, n + 1)</math>, squares <math>a^2, b^2</math>, even numbers <math>2n</math>, and odd numbers <math>2n + 1</math>;</li> <li>• Solve problems involving inverse proportion using graphs, and read values from graphs;</li> <li>• Find the equation of the line through two given points;</li> <li>• Recognise, sketch and interpret graphs of simple cubic functions;</li> <li>• Recognise, sketch and interpret graphs of the reciprocal function <math>y = \frac{1}{x}</math> with <math>x \neq 0</math>;</li> <li>• Use graphical representations of indirect proportion to solve problems in context;</li> <li>• identify and interpret the gradient from an equation <math>ax + by = c</math>;</li> <li>• Write simultaneous equations to represent a situation;</li> <li>• Solve simultaneous equations (linear/linear) algebraically and graphically;</li> <li>• Solve simultaneous equations representing a real-life situation, graphically and algebraically, and interpret the solution in the context of the problem;</li> </ul>	<ul style="list-style-type: none"> <li>• Solve two simultaneous equations in two variables (linear/linear) algebraically and find approximate solutions using a graph.</li> <li>• Identify expressions, equations, formulae and identities from a list.</li> </ul>
Opportunities for reasoning/problem solving	Common Misconceptions
<ul style="list-style-type: none"> <li>• Simple simultaneous equations can be formed and solved from real life scenarios, such as 2 adult and 2 child tickets cost £18, and 1 adult and 3 child tickets costs £17. What is the cost of 1 adult ticket?</li> </ul>	<ul style="list-style-type: none"> <li>• The effects of transforming functions are often confused.</li> </ul>

Notes	Suggested Activities
<ul style="list-style-type: none"><li data-bbox="141 268 840 300">• Emphasise the need for good algebraic notation.</li></ul>	